FAR BEYOND

MAT122

Definite Integral

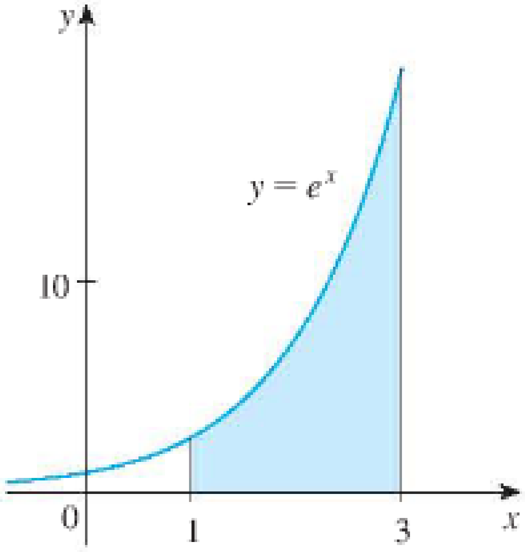
Stony Brook University

Definite Integration Notation

Definite Integral

$$\int_a^b f(x)\,dx$$

$x = a$ and $x = b$ are the bounds of the integration

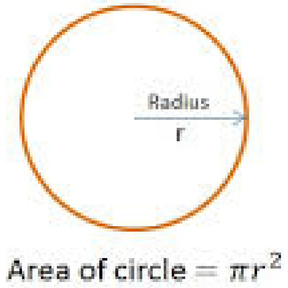


$$\int_1^3 e^x dx$$

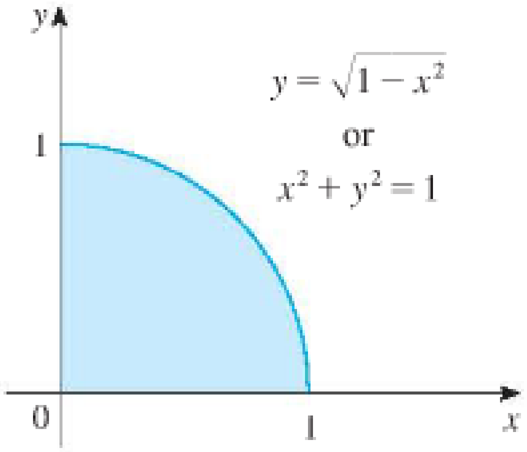
Evaluating Integrals using Area of Circle

ex. Evaluate the following integral *by interpreting in terms of area.*

Area of a Circle:



$$\int_0^1 \sqrt{1-x^2}\,dx$$



$$= \boxed{\frac{\pi}{4}}$$

Indefinite vs. Definite Integral

Recall: $\boxed{\int ax^n dx = \dfrac{ax^{n+1}}{n+1} + C}$

The *indefinite* integral has no bounds.

There is a family of solutions.

ex. $\int 4x^3 dx =$

The ***definite*** integral has bounds from $x = a$ to $x = b$. There is a single solution.

If f is continuous on $a \le x \le b$ and $F'(x) = f(x)$ then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

ex. $\int_1^3 4x^3 dx =$

$= \boxed{80}$

Definite Integral w/Polynomial

$$\int_m^n ax^n\,dx = \left.\frac{ax^{n+1}}{n+1}\right|_m^n$$

ex. Evaluate $\int_0^1 \left(1 + \frac{1}{2}u^4 + \frac{2}{5}u^9\right)du$

$$= \left.\left(u + \frac{1}{2}\cdot\frac{u^5}{5} + \frac{2}{5}\cdot\frac{u^{10}}{\underset{5}{10}}\right)\right|_0^1$$

$$= \left.\left(u + \frac{u^5}{10} + \frac{u^{10}}{25}\right)\right|_0^1 = F(b) - F(a)$$

$$= F(1) - F(0)$$

$$= \boxed{\frac{57}{50}}$$

Definite Integral w/Radical

ex. Evaluate $\int_1^{18} \sqrt{\frac{3}{z}}\, dz$

$$\int_m^n ax^n dx = \left.\frac{ax^{n+1}}{n+1}\right|_m^n$$

$$= \boxed{2\sqrt{3}\left(3\sqrt{2} - 1\right)}$$

Definite Integral with Exponential

$\left(e^x\right)' = e^x$

$\int_m^n e^x dx = e^x \Big|_m^n$

ex. Evaluate $\int_0^1 e^x \; dx$

$= \boxed{e \quad -1}$

ex. Find $\dfrac{d}{dx}\left(\dfrac{e^{3x}}{3}\right)$

$= \boxed{e^{3x}}$

$\int e^u dx = \dfrac{e^u}{u'} + C$

$\int_m^n e^u dx = \dfrac{e^u}{u'}\Bigg|_m^n$

ex. Evaluate $\int \; e^{3x} \; dx$